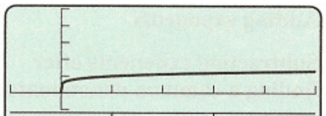
$y_1 = \sqrt{\sqrt[3]{x}},\ y_2 = \sqrt[6]{x}$



X	Y1	Y2
3	1.2009	1.2009
4	1.2599	1.2599
X = 3		

d) $\sqrt{\sqrt[3]{x}} = \sqrt{x^{1/3}}$ — Converting the radicand to exponential notation

$= (x^{1/3})^{1/2}$

$= x^{1/6}$ — Using the laws of exponents

$= \sqrt[6]{x}$ — Returning to radical notation

We can check by graphing $y_1 = \sqrt{\sqrt[3]{x}}$ and $y_2 = \sqrt[6]{x}$. The graphs coincide, as we also see by scrolling through the table of values shown at left.

Try Exercise 91.

7.2 Exercise Set

FOR EXTRA HELP MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Concept Reinforcement *In each of Exercises 1–8, match the expression with the equivalent expression from the column on the right.*

1. (g) $x^{2/5}$ — a) $x^{3/5}$
2. (c) $x^{5/2}$ — b) $(\sqrt[5]{x})^4$
3. (e) $x^{-5/2}$ — c) $\sqrt{x^5}$
4. (h) $x^{-2/5}$ — d) $x^{1/2}$
5. (a) $x^{1/5} \cdot x^{2/5}$ — e) $\dfrac{1}{(\sqrt{x})^5}$
6. (d) $(x^{1/5})^{5/2}$ — f) $\sqrt[4]{x^5}$
7. (b) $\sqrt[5]{x^4}$ — g) $\sqrt[5]{x^2}$
8. (f) $(\sqrt[4]{x})^5$ — h) $\dfrac{1}{(\sqrt[5]{x})^2}$

Note: Assume for all exercises that even roots are of nonnegative quantities and that all denominators are nonzero.

Write an equivalent expression using radical notation and, if possible, simplify.

9. $x^{1/6}$ $\sqrt[6]{x}$
10. $y^{1/5}$ $\sqrt[5]{y}$
11. $16^{1/2}$ 4
12. $8^{1/3}$ 2
13. $32^{1/5}$ 2
14. $64^{1/6}$ 2
15. $9^{1/2}$ 3
16. $25^{1/2}$ 5
17. $(xyz)^{1/2}$ $\sqrt{xyz}$
18. $(ab)^{1/4}$ $\sqrt[4]{ab}$
19. $(a^2b^2)^{1/5}$ $\sqrt[5]{a^2b^2}$
20. $(x^3y^3)^{1/4}$ $\sqrt[4]{x^3y^3}$
21. $t^{2/5}$ $\sqrt[5]{t^2}$
22. $b^{3/2}$ $\sqrt{b^3}$
23. $16^{3/4}$ 8
24. $4^{7/2}$ 128
25. $27^{4/3}$ 81
26. $9^{5/2}$ 243
27. $(81x)^{3/4}$ $27\sqrt[4]{x^3}$
28. $(125a)^{2/3}$ $25\sqrt[3]{a^2}$
29. $(25x^4)^{3/2}$ $125x^6$
30. $(9y^6)^{3/2}$ $27y^9$

Write an equivalent expression using exponential notation.

31. $\sqrt[3]{20}$ $20^{1/3}$
32. $\sqrt[3]{19}$ $19^{1/3}$
33. $\sqrt{17}$ $17^{1/2}$
34. $\sqrt{6}$ $6^{1/2}$
35. $\sqrt{x^3}$ $x^{3/2}$
36. $\sqrt{a^5}$ $a^{5/2}$
37. $\sqrt[5]{m^2}$ $m^{2/5}$
38. $\sqrt[5]{n^4}$ $n^{4/5}$
39. $\sqrt[4]{cd}$ $(cd)^{1/4}$
40. $\sqrt[5]{xy}$ $(xy)^{1/5}$
41. $\sqrt[5]{xy^2z}$ $(xy^2z)^{1/5}$
42. $\sqrt[7]{x^3y^2z^2}$ $(x^3y^2z^2)^{1/7}$
43. $(\sqrt{3mn})^3$ $(3mn)^{3/2}$
44. $(\sqrt[3]{7xy})^4$ $(7xy)^{4/3}$
45. $(\sqrt[7]{8x^2y})^5$ $(8x^2y)^{5/7}$
46. $(\sqrt[6]{2a^5b})^7$ $(2a^5b)^{7/6}$
47. $\dfrac{2x}{\sqrt[3]{z^2}}$ $\dfrac{2x}{z^{2/3}}$
48. $\dfrac{3a}{\sqrt[5]{c^2}}$ $\dfrac{3a}{c^{2/5}}$

Write an equivalent expression with positive exponents and, if possible, simplify.

49. $8^{-1/3}$ $\dfrac{1}{2}$
50. $10{,}000^{-1/4}$ $\dfrac{1}{10}$
51. $(2rs)^{-3/4}$ $\dfrac{1}{(2rs)^{3/4}}$
52. $(5xy)^{-5/6}$ $\dfrac{1}{(5xy)^{5/6}}$
53. $\left(\dfrac{1}{16}\right)^{-3/4}$ 8
54. $\left(\dfrac{1}{8}\right)^{-2/3}$ 4
55. $\dfrac{2c}{a^{-3/5}}$ $2a^{3/5}c$
56. $\dfrac{3b}{a^{-5/7}}$ $3a^{5/7}b$
57. $5x^{-2/3}y^{4/5}z$ $\dfrac{5y^{4/5}z}{x^{2/3}}$
58. $2ab^{-1/2}c^{2/3}$ $\dfrac{2ac^{2/3}}{b^{1/2}}$
59. $3^{-5/2}a^3b^{-7/3}$ $\dfrac{a^3}{3^{5/2}b^{7/3}}$
60. $2^{-1/3}x^4y^{-2/7}$ $\dfrac{x^4}{2^{1/3}y^{2/7}}$

61. $\left(\frac{2ab}{3c}\right)^{-5/6}$ $\left(\frac{3c}{2ab}\right)^{5/6}$

62. $\left(\frac{7x}{8yz}\right)^{-3/5}$ $\left(\frac{8yz}{7x}\right)^{3/5}$

63. $\frac{6a}{\sqrt[4]{b}}$ $\frac{6a}{b^{1/4}}$

64. $\frac{7x}{\sqrt[3]{z}}$ $\frac{7x}{z^{1/3}}$

Graph using a graphing calculator.

65. $f(x) = \sqrt[4]{x+7}$ ⊡

66. $g(x) = \sqrt[5]{4-x}$ ⊡

67. $r(x) = \sqrt[7]{3x-2}$ ⊡

68. $q(x) = \sqrt[6]{2x+3}$ ⊡

69. $f(x) = \sqrt[6]{x^3}$ ⊡

70. $g(x) = \sqrt[8]{x^2}$ ⊡

Approximate. Round to the nearest thousandth.

71. $\sqrt[5]{9}$ 1.552

72. $\sqrt[6]{13}$ 1.533

73. $\sqrt[4]{10}$ 1.778

74. $\sqrt[7]{-127}$ −1.998

75. $\sqrt[3]{(-3)^5}$ −6.240

76. $\sqrt[10]{(1.5)^6}$ 1.275

Use the laws of exponents to simplify. Do not use negative exponents in any answers.

77. $7^{3/4} \cdot 7^{1/8}$ $7^{7/8}$

78. $11^{2/3} \cdot 11^{1/2}$ $11^{7/6}$

79. $\frac{3^{5/8}}{3^{-1/8}}$ $3^{3/4}$

80. $\frac{8^{7/11}}{8^{-2/11}}$ $8^{9/11}$

81. $\frac{5.2^{-1/6}}{5.2^{-2/3}}$ $5.2^{1/2}$

82. $\frac{2.3^{-3/10}}{2.3^{-1/5}}$ $\frac{1}{2.3^{1/10}}$

83. $(10^{3/5})^{2/5}$ $10^{6/25}$

84. $(5^{5/4})^{3/7}$ $5^{15/28}$

85. $a^{2/3} \cdot a^{5/4}$ $a^{23/12}$

86. $x^{3/4} \cdot x^{1/3}$ $x^{13/12}$

Aha! 87. $(64^{3/4})^{4/3}$ 64

88. $(27^{-2/3})^{3/2}$ $\frac{1}{27}$

89. $(m^{2/3}n^{-1/4})^{1/2}$ $\frac{m^{1/3}}{n^{1/8}}$

90. $(x^{-1/3}y^{2/5})^{1/4}$ $\frac{y^{1/10}}{x^{1/12}}$

Use rational exponents to simplify. Do not use fraction exponents in the final answer.

91. $\sqrt[8]{x^4}$ $\sqrt{x}$

92. $\sqrt[6]{a^2}$ $\sqrt[3]{a}$

93. $\sqrt[4]{a^{12}}$ a^3

94. $\sqrt[3]{x^{15}}$ x^5

95. $\sqrt[12]{y^8}$ $\sqrt[3]{y^2}$

96. $\sqrt[10]{t^6}$ $\sqrt[5]{t^3}$

97. $(\sqrt[7]{xy})^{14}$ x^2y^2

98. $(\sqrt[3]{ab})^{15}$ a^5b^5

99. $\sqrt[4]{(7a)^2}$ $\sqrt{7a}$

100. $\sqrt[8]{(3x)^2}$ $\sqrt[4]{3x}$

101. $(\sqrt[8]{2x})^6$ $\sqrt[4]{8x^3}$

102. $(\sqrt[10]{3a})^5$ $\sqrt{3a}$

103. $\sqrt{\sqrt[5]{m}}$ $\sqrt[10]{m}$

104. $\sqrt[4]{\sqrt{x}}$ $\sqrt[8]{x}$

105. $\sqrt[4]{(xy)^{12}}$ x^3y^3

106. $\sqrt{(ab)^6}$ a^3b^3

107. $(\sqrt[5]{a^2b^4})^{15}$ a^6b^{12}

108. $(\sqrt[3]{x^2y^5})^{12}$ x^8y^{20}

109. $\sqrt[3]{\sqrt[4]{xy}}$ $\sqrt[12]{xy}$

110. $\sqrt[5]{\sqrt{2a}}$ $\sqrt[10]{2a}$

TW 111. If $f(x) = (x+5)^{1/2}(x+7)^{-1/2}$, find the domain of f. Explain how you found your answer.

TW 112. Let $f(x) = 5x^{-1/3}$. Under what condition will we have $f(x) > 0$? Why?

⊡ Answers to Exercises 65–70 are on p. IA-15.

SKILL REVIEW

To prepare for Section 7.3, review multiplying and factoring polynomials (Sections 5.3 and 5.6).

Multiply. [5.3]

113. $(x+5)(x-5)$ $x^2 - 25$

114. $(x-2)(x^2+2x+4)$ $x^3 - 8$

Factor. [5.6]

115. $4x^2 + 20x + 25$ $(2x+5)^2$

116. $9a^2 - 24a + 16$ $(3a-4)^2$

117. $5t^2 - 10t + 5$ $5(t-1)^2$

118. $3n^2 + 12n + 12$ $3(n+2)^2$

SYNTHESIS

TW 119. Explain why $\sqrt[3]{x^6} = x^2$ for any value of x, whereas $\sqrt[2]{x^6} = x^3$ only when $x \geq 0$.

TW 120. If $g(x) = x^{3/n}$, in what way does the domain of g depend on whether n is odd or even?

Use rational exponents to simplify.

121. $\sqrt{x\sqrt[3]{x^2}}$ $\sqrt[6]{x^5}$

122. $\sqrt[4]{\sqrt[3]{8x^3y^6}}$ $\sqrt[4]{2xy^2}$

123. $\sqrt[12]{p^2 + 2pq + q^2}$ $\sqrt[6]{p+q}$

124. *Herpetology.* The daily number of calories c needed by a reptile of weight w pounds can be approximated by $c = 10w^{3/4}$. Find the daily calorie requirement of a green iguana weighing 16 lb.
Source: www.anapsid.org 80 calories

Music. The function given by $f(x) = k2^{x/12}$ can be used to determine the frequency, in cycles per second, of a musical note that is x half-steps above a note with frequency k. Use this information for Exercises 125–127.*

125. The frequency of concert A for a trumpet is 440 cycles per second. Find the frequency of the A that is two octaves (24 half-steps) above concert A. (Few trumpeters can reach this note!) 1760 cycles per second

*This application was inspired by information provided by Dr. Homer B. Tilton of Pima Community College East.

126. Show that the G that is 7 half-steps (a "perfect fifth") above middle C (262 cycles per second) has a frequency that is about 1.5 times that of middle C. $2^{7/12} \approx 1.498 \approx 1.5$

127. Show that the C sharp that is 4 half-steps (a "major third") above concert A (see Exercise 125) has a frequency that is about 25% greater than that of concert A. $2^{4/12} \approx 1.2599 \approx 1.25$, which is 25% greater than 1.

128. *Baseball.* The statistician Bill James has found that a baseball team's winning percentage P can be approximated by

$$P = \frac{r^{1.83}}{r^{1.83} + \sigma^{1.83}},$$

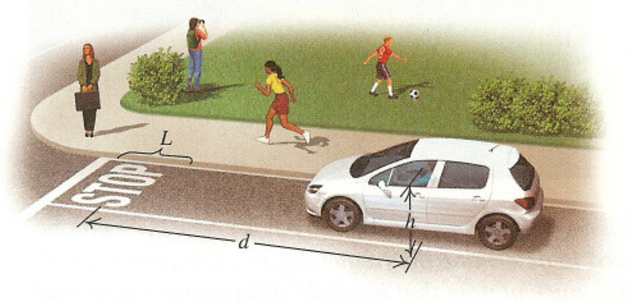
where r is the total number of runs scored by that team and σ is the total number of runs scored by their opponents. During a recent season, the San Francisco Giants scored 799 runs and their opponents scored 749 runs. Use James's formula to predict the Giants' winning percentage. (The team actually won 55.6% of their games.) 53.0%

Source: M. Bittinger, *One Man's Journey Through Mathematics*. Boston: Addison-Wesley, 2004

129. *Road Pavement Messages.* In a psychological study, it was determined that the proper length L of the letters of a word printed on pavement is given by

$$L = \frac{0.000169d^{2.27}}{h},$$

where d is the distance of a car from the lettering and h is the height of the eye above the surface of the road. All units are in meters. This formula says that if a person is h meters above the surface of the road and is to be able to recognize a message d meters away, that message will be the most recognizable if the length of the letters is L. Find L to the nearest tenth of a meter, given d and h.



a) $h = 1$ m, $d = 60$ m 1.8 m
b) $h = 0.9906$ m, $d = 75$ m 3.1 m
c) $h = 2.4$ m, $d = 80$ m 1.5 m
d) $h = 1.1$ m, $d = 100$ m 5.3 m

130. *Physics.* The equation

$$m = m_0(1 - v^2c^{-2})^{-1/2},$$

developed by Albert Einstein, is used to determine the mass m of an object that is moving v meters per second and has mass m_0 before the motion begins. The constant c is the speed of light, approximately 3×10^8 m/sec. Suppose that a particle with mass 8 mg is accelerated to a speed of $\frac{9}{5} \times 10^8$ m/sec. Without using a calculator, find the new mass of the particle. 10 mg

131. *Forestry.* The total wood volume T, in cubic feet, in a California black oak can be estimated using the formula

$$T = 0.936d^{1.97}h^{0.85},$$

where d is the diameter of the tree at breast height and h is the total height of the tree. How much wood is in a California black oak that is 3 ft in diameter at breast height and 80 ft high?

Source: Norman H. Pillsbury and Michael L. Kirkley, 1984. Equations for total, wood, and saw-log volume for thirteen California hardwoods, USDA Forest Service PNW Research Note No. 414: 52 p. 338 cubic feet

132. A person's body surface area (BSA) can be approximated by the DuBois formula

$$\text{BSA} = 0.007184w^{0.425}h^{0.725},$$

where w is mass, in kilograms, h is height, in centimeters, and BSA is in square meters. What is the BSA of a child who is 122 cm tall and has a mass of 29.5 kg? Approximately 0.99 m^2

Source: www.halls.md

133. Using a graphing calculator and a $[-10, 10, -1, 8]$ window, select the MODE SIMUL and the FORMAT EXPROFF. Then graph

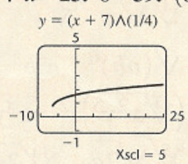
$$y_1 = x^{1/2}, \quad y_2 = 3x^{2/5},$$
$$y_3 = x^{4/7}, \quad \text{and} \quad y_4 = \frac{1}{5}x^{3/4}.$$

Looking only at coordinates, match each graph with its equation.

Try Exercise Answers: Section 7.2

9. $\sqrt[6]{x}$ **23.** 8 **39.** $(cd)^{1/4}$ **43.** $(3mn)^{3/2}$ **49.** $\frac{1}{2}$

65.



71. 1.552 **77.** $7^{7/8}$ **91.** $\sqrt{x}$